Symposium on Harvest and Post Harvest Technology of Fish held during 24-27 November 1982 at Cochin,389-391.

Transportation of Fresh Fish : Mathematical Models for the Expenditure Pattern

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Taking into account the main factors of expenditure in the transportation of fish from one centre to another, mathematical models are developed for iced, frozen and ice/re-iced (enroute) fish for calculation of expenditure (in the transportation). These models help us to arrive at the average expenditure involved under different sets of conditions so that the most economical methods of transportation under given conditions can be chosen.

Transportation of fresh fish over long distances by different modes of transport has been extensively studied under an All India Coordinated Research Project (1971–79) undertaken by the Indian Council of Agricultural Research with its coordinating centre at Central Institute of Fisheries Technology, Cochin. Transportation of fish in ice, with or without reicing enroute at transhipment points, or of frozen fish etc.. have been subjected to detailed study. A new thermocole lined container very efficient for this purpose has been developed and extensively used in field studies. The results of these studies have been summarised in the final report of the project and a detailed account of the container published as a hand out (Anon, 1981).

One of the important considerations in transportation of fish is the economic aspect. Procedure for calculating the average expenditure for transportation of unit quantity of fish under different conditions is provided in this paper with mathematical models.

Notations used

The following notations are used in working out the mathematical models.

Capacity of the container	—	'K'	kg
Quantity of iced fish sent in a container	—	'X'	kg
Quantity of frozen fish packed in container	—	'Y'	kg
Quantity of ice used for one kg of fish	—	'a'	kg
Quantity of unmelt ice in the container kg^{-1} of fish at the reicing point in the intermediate station	—	'a^1'	kg
Additional weight for one kg of frozen fish due to glaze etc.	—	'b'	kg
Weight of container with insulation material but without fish	—	'W'	kg
Cost of one kg of ice at the transporting centre	—	'p_1'	Rs.
Cost of one kg of ice at the intermediate centre (where re-icing is involved)	—	'$p_1{}^1$'	Rs.
Cost of freezing one kg of fish	—	'p_2'	Rs.
Cost of container with the insulation material etc. (Proportionate cost where applicable)	—	'C'	Rs.
Freight and handling charges from the transporting centre to the consuming centre, per kg	—	'f'	Rs.
Freight and handling charges from the transporting centre to the intermediate station, per kg	—	'f_1'	Rs.
Freight and handling charges from the intermediate station to the consuming centre	—	'f_2'	Rs.
(f_1 and f_2 are applicable where re-icing at an intermediate station is involved)			
Total expenditure on transportation	—	'T_1'	Rs.
Average expenditure, per kg of fish on transportation	—	'T_2'	Rs.

Mathematical models

(i) *Iced fish*

The total expenditure includes the cost of the container, icing, freight and handling charges. This can be shown to be

$$T_1 \text{ (iced)} = C + a \times p_1 + f \left\{ X(1+a) + W \right\} \qquad (1)$$

From the above, the average expenditure per kg of fish will be

$$T_2 \text{ (iced)} = \frac{C}{X} + ap_1 + f \left\{ (1+a) + \frac{W}{X} \right\} \qquad (2)$$

As in the case of iced fish, this can be modified when the fulll capacity of the container is utilised by substituting $X = \frac{K}{1+a}$. In that case we obtain:

$$T_2 \text{ (iced/re-iced)} = \frac{C(1+a)}{K} + a\,(p_1 + p_1{}^1) + (f_1 + f_2) \left\{ (1 + a) + W \frac{(1 + a)}{K} \right\} = (1 + a) \left\{ \frac{C}{K} + (f_1 + f_2) (1 + \frac{W}{K}) \right\} = a\,(p_1 + p_1{}^1) \quad (8)$$

The above equations are on the presumption that the unmelted portion of ice in the container at the intermediate station is discraded and full quantity of required fresh ice is put into the container. If however the unmelted portion of ice is not removed and only the balance quantity of required ice is added, then the equations will be as follows:

$$T_1 \text{ (iced/re-iced)} = C + X\,(ap_1 + ap_1{}^1 + a^1 p_1{}^1 + (f_1 + f_2) \left\{ X(1 + a) + W \right\} \quad (9)$$

and

$$T_2 \text{ (iced/re-iced)} = \frac{C}{X} + (ap_1 + ap_1{}^1 - a^1 p_1{}^1) + (f_1 + f_2) \left\{ (1 + a) + \frac{W}{X} \right\} \quad (10)$$

$$\text{or } T_2 \text{ (iced/re-iced)} = (1 + a) \left\{ \frac{C}{K} + (f_1 + f_2)(1 + \frac{W}{K} \right\} + (ap_1 + ap_1{}^1 - a^1\, p_1{}^1) \quad (11)$$

Equation (6), (7), and (8) can be seen to be special cases of (9), (10) and (11) respectively when $a^1 = 0$

Application

We take the following values of variables $p_1{}^1$, f_1 and f_2 and the rest of the variables with values as stated earlier.

$p_1{}^1 = 0.30,\ f_1 = 0.35,\ f_2 = 0.30$

Here we consider the case when the unmelted ice at the intermediate station is discarded. Thus

$$T_2 \text{ (iced/re-iced)} = \frac{25}{50} + 1.0\ (0.20 + 0.30) + (0.35 + 0.30)\ (2 + \frac{5}{50}) = 0.50 + 0.50 + 1.36 = 2.36$$

Thus Rs. 2.36 will be the average expenditure in transporting one kg of fish under the above conditions. If we choose to make full use of the container for the same value of 'a' we obtain

$$T_2 \text{ (iced/re-iced)} = 2 \left\{ \frac{25}{110} + (0.30 + 0.35)\ (1 + \frac{5}{110}) \right\} - 1.0\ (0.20 + 0.30) = 0.46 + 1.36 + 0.50 = 2.32$$

Thus there is a reduction of Re. 0.04 in the average expenditure when the capacity of the container is fully utilised.

It must be remembered that the values of f, f_1 and f_2 are to be determined in accordance with the prevailing freight charges. The present rates per quintal are Rs. 24.50 for 500 km, Rs. 52.25 for 1500 km and Rs. 85.25 for 3000 km.

The values given for f, f_1 and f_2 in the calculations are per kilogram and the difference accounts for the handling charges.

Long distance transportation: An example of comparison of costs

The above mathematical models can be used to work out cost of transportation of fish under certain special circumstances. Consider a case where fish is to be transported over a distance of 3000 km involving a journey of more than two days. If the consignment has to be sent directly (without re-icing at intermediate stations), it requires higher proportion of ice, 2 kg of ice for 1 kg of fish. This will allow only far less quantity of fish to be packed but has the advantage of single handling and less freight charges. If the fish is re-iced midway, say at 1500 km from the starting point, ice requirement is less (1 kg of ice for 1 kg of fish) which provides for packing more fish. However, the consignment has to be booked in two stages, adding to the freight and handling expenses. A third alternative is to send as frozen fish, but freezing is much more expensive than icing. The least expensive of the three methods has to be determined. The equations given earlier help to work out the comparative costs of transportation. In arriving at the average expenditure, take $f = 0.90$ and $f_1 = f_2 = 0.60$. It is assumed that the full capacity of the container is made use of in all the three cases. The value of 'a' is taken as 2.0 for direct booking and 1.0 when re-iced at intermediate station. The optimum quantity of fish in frozen condition that can be transported in a container of 110 kg capacity is taken as 80 kg. The values of all other variables are taken as mentioned under the different examples. By substituting the appropriate values and simplifying, we obtain the following values.

T_2 (iced)	=	Rs. 3.91
T_2 (iced/reiced)	=	Rs. 3.47
T_2 (frozen)	=	Rs. 2.54

Instead of choosing a particular quantity of fish, 'X' kg, if we wish to make use of the full capacity of the container, we have

$$X(1 + a) = K,\text{ie. } X = \frac{K}{1+a}$$

Substituting the above value of 'X' in (2), we obtain

$$T_2 \text{(iced)} = C\,(\frac{1+a}{K}) + ap_1 + f \left\{ (1 + a) + W\,(\frac{1+a}{K}) \right\} = a \left\{ \frac{C}{K} + p_1 + f\ (1 + \frac{W}{K}) \right\} + \left\{ \frac{C}{K} + f(1 + \frac{W}{K}) \right\} \quad (3)$$

(3) is a linear equation in 'a' and is independent of x. From this equation, we can obtain the average expenditure directly for different proportions of fish to ice, when we make use of the full capacity of the container.

Application

We take the following values for the different variables.

$X=50$ $p^1=0.20$
$a=1.0$ $C=25.00$
$W=5$ $f=0.27$

Substituting in (2), we have

$$T_2 \text{ (iced)} = \frac{25}{50} + 0.20 + 0.27\left(2 + \frac{5}{50}\right)$$
$$= 0.50 + 0.20 + 0.57 = 1.27$$

Thus the average expenditure for the above values of the variables is 1.27 Rs. kg^{-1} of fish.

Instead of x = 50, if we make use of the full capacity of the container with a capacity of 110 kg, we have K = 110 and substituting all the above values, except 'a', in (3) and simplifying, we get the equation.

$$T_2 = 0.7095\,a + 0.5095$$

The above is a linear equation in 'a' and shows that for every additional kg of ice used for one kg of fish, the average additional expenditure will increase by 0.71 Re. for the values cf the variable chosen here. In this connection, it has to be remembered that if the quantity of ice in the container is increased the quantity of fish that can be packed in the container has to be correspondingly reduced. The above equation takes care of that aspect.

(ii) *Frozen fish*

In the case of frozen fish, the total expenditure can be shown to be

$$T_1 \text{ (frozen)} = C + Yp_2 + f\left\{Y\ (1+b) + W\right\} \qquad (4)$$

and the average expenditure will be

$$T_2 \text{ (frozen)} = \frac{C}{Y} + p_2 + f\left\{(1+b) + \frac{W}{Y}\right\} \qquad (5)$$

with the notations explained already.

As in the case of iced fish, we can work out T_1 and T_2 when the full capacity of the container is utliized. In the case of iced fish the proportion 'a' is a variable, but in the case of frozen fish 'b' has more or less a fixed value representing the additional weight of the frozen fish for one kg of fish. The maximum quantity of frozen fish that can be put in a container of given capacity will be fixed and that value has to be used for Y in (4) and (5).

Application

Let us assume the following values for variables Y, b and p_2.

$Y = 50$
$b = 0.3$
$p_2 = 1.00$

The value of the rest of the variables figuring in (5) will be the same as for iced fish. With these, we obtain

$$T_2 \text{ (frozen)} = \frac{25}{50} + 1.00 + 0.27\,(1.3 + 0.10)$$
$$= 0.50 + 1.00 + 0.38 = 1.88 \text{ Rs.}$$

Thus the average expenditure in transporting one kg of fish is Rs. 1.88.

If on the other hand, the full capacity of a container of capacity 110 kg is to be utilised, we can assume the quantity of fish needed in fresh condition, to be 80 kg. Thus susbstituting Y = 80 in (5) and simplifying, we obtain T_2 (frozen) = 1.68 Rs. which is reduction of Rs. 0.20

(iii) *Iced fish, re-iced enroute*

With the earlier notations, the total expenditure can be shown to be

$$T_1 \text{ (iced/re-iced)} = C + aX\,(p_1 + p_1^1)\,(f_1 + f_2)\left\{X\ (1+a) + W\right\} \qquad (6)$$

and the average expenditure per kg of fish

$$T_2 \text{ (iced/re-iced)} = \frac{C}{X} + a\ (p_1 + p_1^1) + (f_1 + f_2)\left\{(1+a) + \frac{W}{X}\right\} \qquad (7)$$

This shows that under this specific circumstance the cost of transportation is the cheapest when frozen fish is used. Iced and reiced fish is more economical than iced fish in this case. Transportation of iced fish is the most expensive, more than, one and half times than that of transporting frozen fish.

References

Anon (1979) *Final report of Research Scheme, All India Co-ordinated Research Project on Transporation of Fresh Fish and Utilization of Trash Fish.* ICAR. CIFT, Cochin

Anon (1981) *Improved Container for Transporation of Fresh Fish,* Central Institute of Fisheries Technology, Cochin, India.